RESEARCH ARTICLE

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Reasoning Processes of Prospective Teachers in Proving Theorem Regarding Differences in Mathematical Ability

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The aim of this study is to describe reasoning processes of prospective teachers in proving theorems about similarity of triangles regarding differences in mathematical ability. Research subjects are college students from undergraduate students of Mathematics Education. The subjects consisted of three students who have high, medium, and low mathematical ability, and have programmed Geometry courses. Reasoning processes of prospective teachers in proving theorem was derived from task-based interviews. The reasoning of prospective teachers' mathematical ability of high, medium, and low in the understanding theorem is similar. When choosing a strategy, prospective teachers with high and medium mathematical ability have similarity in predicting, that there are three ways in proving the theorem but prospective teacher of low mathematical ability chose only a strategy. At the implementing strategy phase, there are similarities between high and low ability students in proving a theorem, although students with high ability used more strategy. A prospective teacher with medium mathematical ability did not prove the theorem using the plan mentioned before. The reasoning of prospective teachers' mathematical ability of high and low when looking back phase are similar, while prospective teachers' mathematical ability of medium is not similar to with the others because there are inconsistent with what has been said. There are similar looking back work of students capable of high and low, while students capable of being which is said to be consistent.

Keywords: Reasoning, Prospective Teachers, Proving Theorem, Mathematical Ability.

1. INTRODUCTION

Thinking is a process in which new mental representation is created through the transformation of information with the complex interactions of mental attributes that can be inferred from behavior that includes consideration, classification, interpretation, judgment, abstraction, reasoning, imagery, and problem solving.[1] Therefore, if someone thinks, it is not necessary that he/she does reasoning. However, if a person does reasoning, he/she would think too. Logical thinking is a thinking based on logic. Lohman and Lakin[2] state that "reasoning refers to the process of drawing conclusions or inferences from information." Longman[3] states that "reasoning is the ability to think, understand, and form opinions or judgments that are based on facts. It is the process of making inferences from a body of information." The reasoning is a logical thinking to get conclusion based on the truth has been proven or based on premises or information provided.[4]

It is now generally recognized that reasoning is fundamental to understand and apply mathematics, and that mathematical reasoning should be fostered by engaging students in investigating, representing, conjecturing, explaining, and justifying mathematics.[5,6] Studying students' skills and abilities of mathematical reasoning is not an easy task because, generally speaking, this process is influenced by educational, cultural, social and historical approaches.[7] Mathematics reasoning is emphasized in many mathematics contexts, but the results of research conducted by Reyhani et al.[7] showed that many students experience difficulty in the study and write theorem proof by using mathematical reasoning. Portnoy[8] argues that introduce reasoning and proof as fundamental mathematics aspects to students is challenge for future teacher and teacher education programs.

One activity that shows a person's reasoning ability is proving theorems. Basically proving is an activity to demonstrate the truth of a statement with a set of rational arguments. Tall and Mejia-Ramos, Hanna and Harel and Sowder (in Ref. [9]) state that "proofs supply correctness and incorrectness of every situation in mathematics. Not only proofs show whether a situation is correct or wrong, but also proofs show why it is wrong or correct. Proving is defined as a mental action used for removing doubts about any claim mentioned by a society or an individual." In mathematics, reasoning and proof are very closely related.

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Healy and Hoyles (cited in Ref. [10]) state that "Proof is at the heart of mathematical thinking and deductive reasoning."

One of the subjects with full of theorem discussions is geometry. In studying Geometry, students are expected to achieve the following skills:
(1) recognize reasoning and proof as fundamental aspects,
(2) create and investigate a conjecture of mathematics,
(3) develop and evaluate mathematical arguments and evidence, and
(4) select and use different types of reasoning and methods of proof.[6]

There are three major reasons geometry is taught in schools and in universities, namely (1) expand the spatial awareness, (2) develop reasoning skills and (3) to stimulate, challenge and inform learners. According to Larson et al.[11] there are three main aspects that are studied in geometry that is the measurement, reasoning, and application.

Problem-solving is an example of its application, and by Polya,[12] problems can be differentiated into the problem to find and problem to prove. The problem to find is a type of problem that the goal will be found, and the process is required. The problem to prove is a problem that goal is determined but the process is required. In this study, the reasoning process in proving theorem views based on steps (1) understanding the theorem, (2) strategy choice (3) Strategy Implementation (4) Looking back.

Students difficulties in proving (theorem) due to lack of understanding concept. They do not know how to use reasoning and proving language, unable to express their thoughts and lack of specific knowledge about definitions, theorems, and heuristics, as well as the lack of ability to give examples. It shows that in proving (theorem), there is a link between conception knowledge (mathematics) and reasoning since proving theorem require thinking process that involves cognitive systems that are turned to generate ideas based on the premises or information that has been given.

Mathematics is the basic knowledge and ability required to perform mathematical manipulations include conceptual understanding and procedural knowledge. Things that include conceptual understanding are reasoning ability, identity, and apply principles, ability to manipulate ideas about conceptual understanding in various ways, and others. Therefore, if a person has good reasoning ability, conceptual understanding is expected to be good too. Thus, college students who used to use deductive reasoning regularly will help them learn the basic deductive principles (mathematics), so it will be easier for them to learn. The creative mathematical reasoning is not only owned by highly qualified people only. This is due to many factors, including environment, situation, and prior knowledge. This is because many factors affect the environment, among others, the situation and the ability of the initial or basic knowledge of students. The statement of the results of research conducted by Siswono[13] to the students of Department of Mathematics Universitas Negeri Surabaya showed that students in three groups (math abilities: high, medium and low) have different strategies to construct and prove theorems. Reyhani et al.[7] state that "Although the proof is one of the basic concepts in mathematics, especially, in mathematics education, many research studies have indicated that students have difficulty when making mathematical proofs."

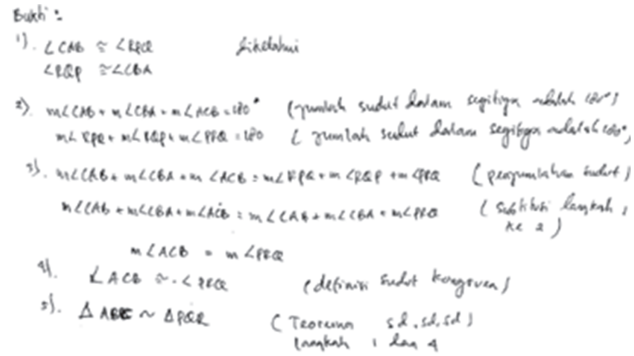

Fig. 1. *Tg*'s task 1.

Based on the above description, the aim of this study is to describe students' reasoning in proving the theorem with differences in math abilities of high, medium and low.

2. METHOD

The type of this research is explanatory research with a qualitative approach. The primary instrument in this research is the researcher herself, while the secondary instruments are: (a) Mathematical Ability Test, (b) Theorem Proving Task, (c) Interview Guide. Mathematical Ability Test score and GPA of Mathematics subject obtained in the first and second semester is used to determine the groups of students with high, medium, and low mathematical abilities. Research subjects are undergraduate students from Mathematics Education. The subjects consisted of three students who have high, medium, and low mathematical ability, and have programmed Geometry courses. Reasoning profile of students in proving theorems were derived from task-based interviews. Data credibility were taken by time triangulation. Data which are credible were analyzed using qualitative research data analysis model including data reduction, data display, data interpretation and conclusion drawing.

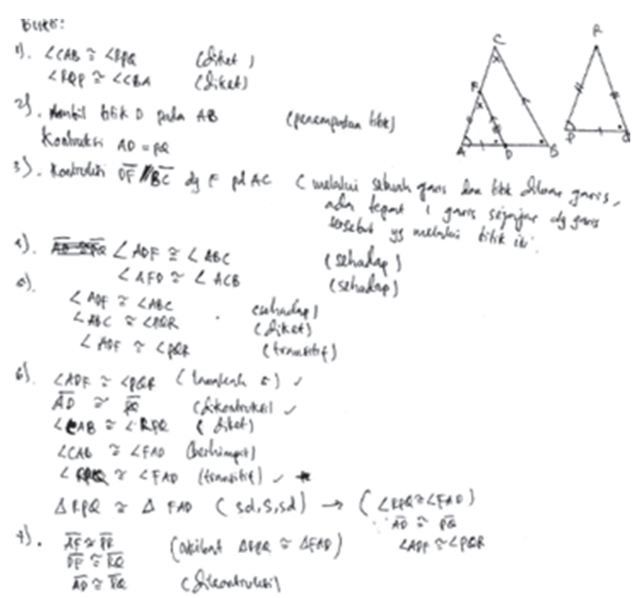

Fig. 2. *Tg*'s task 2.

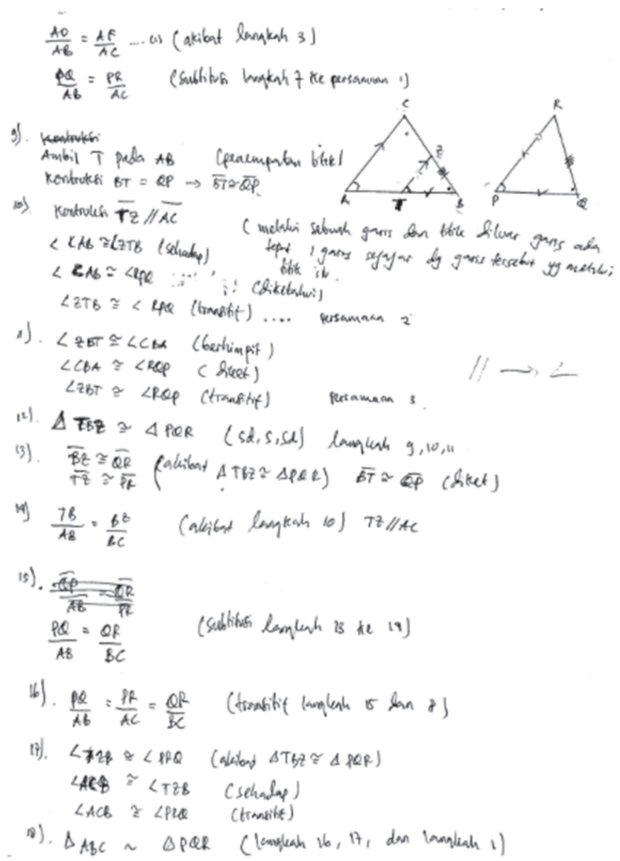

Fig. 3. *Tg*'s task 3.

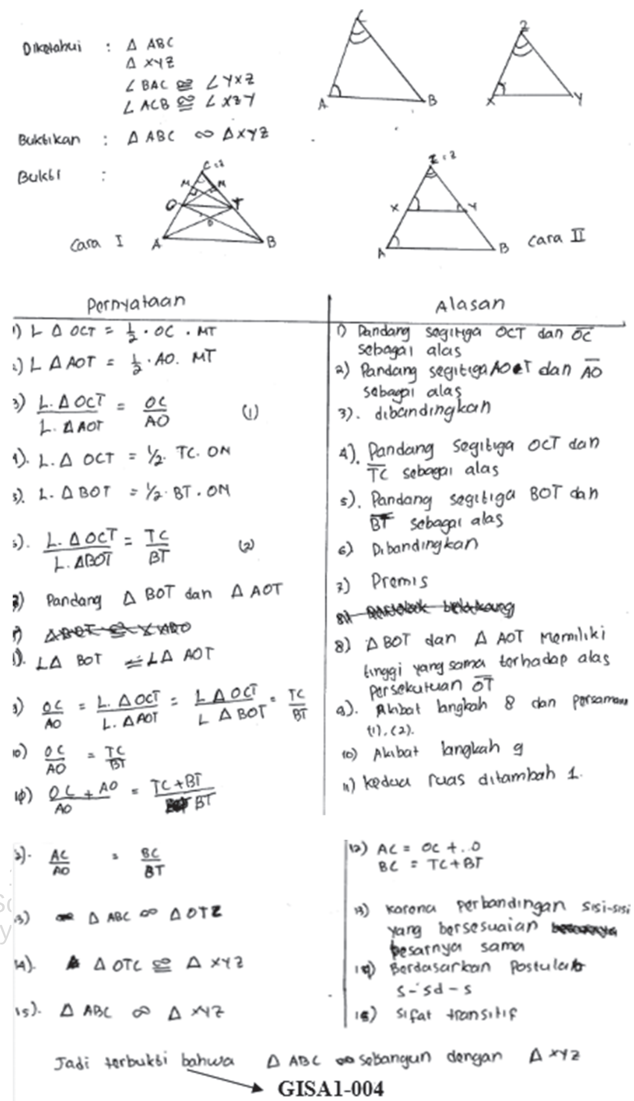

Fig. 4. *Sd*'s task 1.

3. RESULTS AND DISCUSSION

Based on task-based interview data triangulation in proving the theorem "If two triangles have two congruent angles, then the two triangles are similar," the collected data have been declared valid. The analysis of the subjects' reasoning data in proving theorem is described as follows.

3.1. Understanding Theorem

At the stage of understanding the theorem, the three subjects have understood theorem that they have read. They are able to mention what is known and what is to be proved in the theorem. They mention the known or the premise of the theorem that two triangles have two congruent angles with a reason that the theorem has an implication form; that the sentence after the word "if" and before the word "then" is a premise or a known. Furthermore, they mention what is to be proved or the conclusion of the theorem; that is, two triangles are similar because the sentence is written after the word "then" in the theorem.

Because the reasoning of the three subjects said the premise and the conclusion of each theorem is based on rational arguments, the reasoning carried out by three subjects in understanding the theorem is valid or in accordance with the rules of logic.

3.2. Strategy Choice

The subjects with high-ability (Tg) and medium ability (Sd) selected three strategies. The first way was to use direct proof. Tg reasoned using direct proof for the information that will be used to prove is already obvious in the premise. Sd said that direct proof that he would use because usually when evidence is not directly use the right one for this theorem is not mentioned exactly one word it would prove the direct evidence. The second way that will be used to prove the theorem is triangles similarity (Angle–Angle–Angle). Tg showed that the three corresponding angles in two triangles must be congruent by using sum of three interior angles in a triangle is 180 degrees. The third way is using the definition of similarity of two triangles. But to show a comparison of the length of the triangle sides are equal, there is a difference between Tg and Sd. Tg constructed congruent triangles with one triangle that is known in the other triangle which assumed greater in size than the triangle are known. Tg constructed the triangle because the three corresponding angles must be congruent and three sides which are corresponding from the two triangles have the same proportions. While Sd used the concept of the triangle area to show a comparison of the sides' length of the two triangles. Low mathematical ability subject (Rd) choose a strategy to prove theorems by guessing how to used the definition of congruency of two triangles to construct congruent triangles with one triangle that is known in

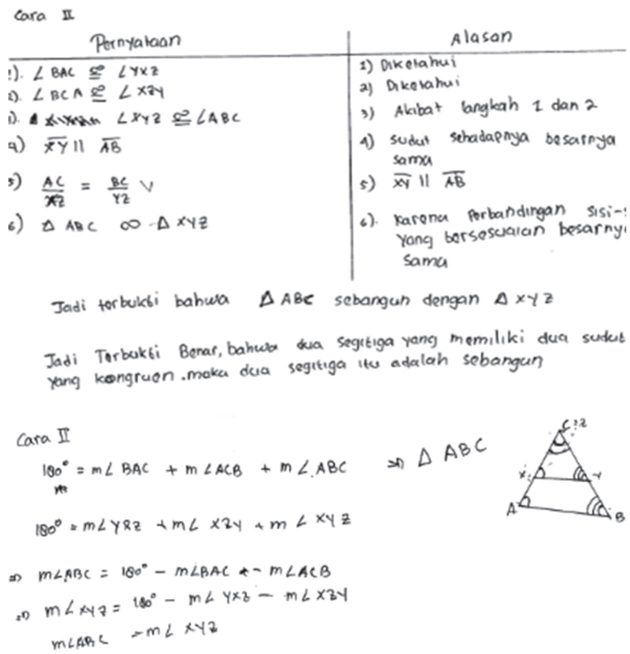

Fig. 5. *Sd*'s task 2.

the other triangle which assumed greater in size than the triangle are known. Sd said that Sd used area concept, congruency, and transitive nature are used so it would prove that those two triangles were similar.

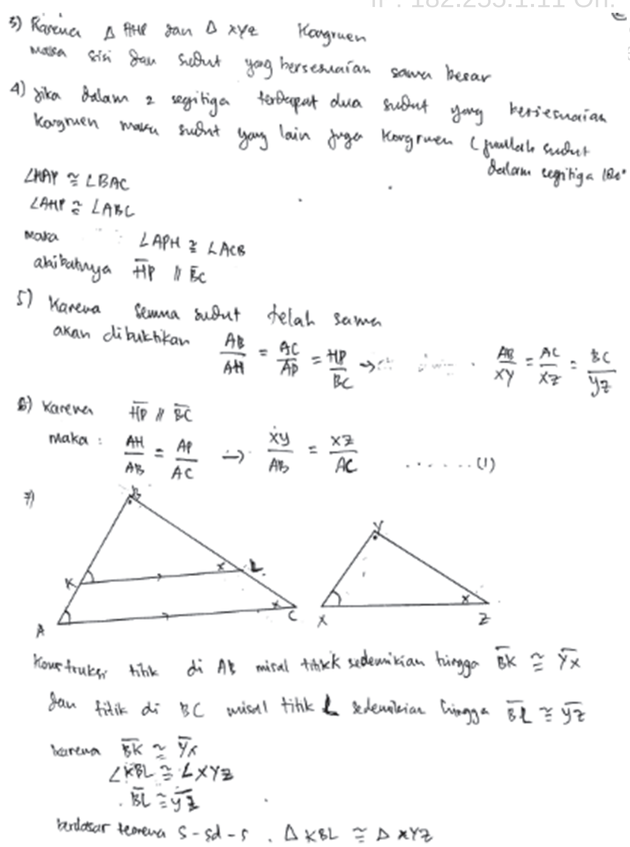

Fig. 6. *Rd*'s task 1.

3.3. Strategy Implementation

3.3.1. Interview Data of a Subject with High Mathematical Ability (Tg) Based Task in Implementing Strategy Phase

3.3.1.1. The First Method Used to Prove Theorems Tg is the Theorem Angle-Angle-Angle Similarity. By using algebraic and geometry manipulation, Tg showed third angle of the two triangles are congruent. Previously, Tg constructed two triangles and named it ΔABC and ΔPQR, with $\angle CAB \cong \angle RPQ$ and $\angle CBA \cong \angle RQP$. Furthermore, by using algebraic manipulations, Tg showed that $\angle ACB \cong \angle PRQ$. The reason given by Tg was theorem about the sum of three interior angles of the triangles, which reads the sum of three interior angles of the triangles is 180 degrees. So that if two angles are known congruent, then the third angles are congruent too. According to the Angle-Angle-Angle Similarity Theorem, Tg proved that $\Delta ABC \sim \Delta PQR$.

3.3.1.2. The Second Way is Used to Prove Theorems Tg with the Definition of Congruency of Two Triangles. Tg had to show the third pair of corresponding angle on the two triangles were congruent and corresponding sides were all in the same proportion.

Furthermore, Tg showed that the corresponding sides were all in the same proportion by constructing D so that $\underline{DF} \| \underline{BC}$ and T so that $\underline{TZ} \| \underline{AC}$. The reason given by Tg is a theorem about parallel line, that was "through a point outside a given line, can be made a line parallel to the given line." Using the theorem about corresponding angles on two parallel lines and the Mid-Segment Theorem, Tg can show that $\Delta ABC \sim \Delta PQR$ based on transitive properties. Because the three pairs of corresponding angle from the triangles were congruent, $\angle CAB \cong \angle RPQ$, $\angle CBA \cong \angle RQP$, and $\angle ACB \cong \angle PRQ$, the corresponding sides were all in the same proportion that $PQ{:}AB = PR{:}AC = QR{:}BC$ also, Tg concluded that the two triangles were similar. Tg also said if the ratio value of the corresponding sides from two triangles was one, so that two triangles were congruent. Both triangles were still similar triangles. This show that the Tg also understand if two triangles are congruent, the two triangles are also similar.

Based on the description above, concluded that the Tg's reasoning process in the step of carry out the plan in proving the theorem has been in accordance with the rules of logic or compliance with coherence theory of truth. This is shown for any statements made by Tg in proving the theorem, Tg can give reasons based on the information available on premise or what have known from the theorem. Tg can also utilize existing information on premise and can connect the concepts, definitions, and theorems related in proving two triangles are similar.

3.3.2. Interview Data of Subject with Medium Mathematical Ability (Sd) Based Task in Implementing Strategy Phase

3.3.2.1. At the Phase of Implementing the Strategy, Sd Used the Area Concept of Triangle to Prove the Theorems. By using algebraic manipulations, Sd compared the two sides of the same triangle. Reasoning Sd in implementing a strategy to prove the theorems was as follows: the first step, Sd drew and let two triangles that were known and named ΔABC and ΔXYZ, giving the same mark on $\angle BAC$ and $\angle YXZ$ so that $\angle ACB$ and $\angle XZY$, the reason was because it was known there were two angles congruent. The next step, Sd constructed point O on the side $\underline{AC}$, point T on the side $\underline{BC}$ and connected O and T such that there was segment $\underline{OT}$. The reason was because on the side $\underline{AC}$ and side $\underline{AB}$ consists of a set of points, it could take on any point on the side $\underline{AC}$ and the side $\underline{AB}$ and connected it. When researcher ask, "Why you write the area ΔOTC and the area ΔAOT?" The answer was the area formula of the triangle that used is $1/2 \times$ number of base $\times$ number of high because ΔOTC and ΔOTC have the same height. So, if the area of ΔOTC and the area of ΔAOT compared than it can be simplified. Furthermore, Sd raised the area of ΔOCT and the area of ΔBOT because analogous to the previous way, if the area of ΔOCT and the area of ΔBOT compared, than it could be simplified because it had the same element. Furthermore, Sd wrote the area of $\Delta BOT =$ the area of ΔAOT, the reason was because ΔBOT and ΔAOT have the same height to base $\underline{OT}$, so $OC/AO = TC/BT$, the next reason was because $OC/AO =$ area of ΔOCT/area of ΔAOT and area of ΔOCT/area of $\Delta AOT = TC/BT$ from equation two, as well as the area of $\Delta BOT =$ the area of ΔAOT, so $OC/AO =$ area of ΔOCT/area of $\Delta AOT =$ area of ΔOCT/area of $\Delta BOT = TC/BT$ it mean that $OC/AO = TC/BT$. Furthermore, Sd add one to both sides of $OC/AO = TC/BT$, so in step 12 is obtained $OC/AO = BC/BT$. If $OC/AO = TC/BT$ mean that the ratio of the corresponding sides of the triangle *ABC* and the triangle *OCT* were equal. Furthermore, Sd wrote step 13 is $\Delta ABC \sim \Delta OTC$, the reason was based the theorem that state side-angle-side, the corresponding sides with same length were $AC/AO = BC/BT$ and angle $\angle OCT$ flanked by side $\underline{OC}$ and side $\underline{TC}$. In triangle ABC and triangle OCT also flanked by side $\underline{AB}$ and side $\underline{TC}$ based theorem side-angle-side. Two triangles could be said similar if the corresponding sides had the same length and the angle that flanked by both these sides had the same great. Furthermore, I squeezed angle $\angle XZY$ to the triangle *ACB*, so angle $\angle XZY$ and angle $\angle ACB$ was equal. However, when researchers asked "whether the length of $\underline{OC}$ is equal with the length of $\underline{XZ}$?," Sd said no, but I was letting *OC* had same length with *XZ* and *CT* equal with *ZY* then we could determine that the two triangles were congruent by side-angle-side. Two triangles were congruent if the corresponding sides had the same length and the angle that flanked by the sides had the same great, ΔOTC and ΔABC had same length ratio while ΔOTC and ΔXYZ had the same length. After that, if you already known that triangle *XYZ* and triangle *OTC* were congruent, the next step was the triangle *ABC* was similar with the triangle *XYZ* because transitive properties.

3.3.2.2. Second Method Used by Sd to Prove Theorem is Side-Angle-Side Similarity. Reasoning that used by Sd that presented already according to the rules of logic. This is because when Sd prove two triangles similar using a large amount of angles in a triangle 180 degrees. The reason given by Sd was explained while increasing write the answers on the triangle *ABC*, $m\angle BAC + m\angle ACB + m\angle ABC = 180$, while the triangle ΔXYZ, $m\angle YXZ + m\angle XZY + m\angle XYZ = 180$, because $m\angle BAC = m\angle YXZ$ and $m\angle ACB = m\angle XZY$ and if 180 was reduced by the same number it means $m\angle ABC = m\angle XYZ$. Furthermore, because angle with the same direction have the same large means that $\underline{XY} \| \underline{AB}$, then because $\underline{XY} \| \underline{AB}$ caused $AC/XZ = BC/YC$. But when asked "why write if $\underline{XY} \| \underline{AB}$ then $AC/XZ = BC/YC$, Sd said because $\underline{XY} \| \underline{AB}$ without being able to explain it again. Furthermore, Sd wrote step 6 is $\Delta ABC \sim \Delta XYZ$, the reason was based the theorem that stateside-angle-side, two triangles could be said similar if the corresponding sides had the same length and the angle that flanked by both these sides had the same great, then ΔABC and ΔXYZ similar, because $AC/XZ = BC/YC$, and angle that flanked by angle C and the angle Z was equal.

Based on the argument above, it can be concluded that the process of reasoning that do by Sd at the phase implementing of strategy that chosen strategy in proving the theorem does not correspond with the theory of coherence truth. This is shown because every statement that made in proving the theorem, the reason given by Sd is not based on information known to the theorems, definitions, and theorems relating that used in the proof.

3.3.3. Interview Data of Subject with Low Mathematical Ability (Rd) Based Task in Implementing Strategy Phase

Rd used the definition of triangle similarity to prove the theorem on stage doing the strategy, Rd's reasoning in doing the strategy first was drawn and presuppose two triangles that known and give name ΔABC and ΔXYZ and also constructed $\Delta AHP \cong \Delta XYZ$ in ΔABC by using theorem side-angle-side. So that Rd got three pairs of corresponding angles in ΔABC and ΔXYZ are congruent and $XY/AB = XZ/AC$ had been corresponding with rule of logic. This is because the reason that given by Rd that because a large number of angles in triangle is 180°, so if two angles are congruent then the third angle is congruent too, because $\Delta AHP \cong \Delta XYZ$ then corresponding angle are congruent, because $\Delta ABC \cong \Delta XYZ$ and $\Delta AHP \cong \Delta XYZ$ so $\Delta AHP \cong \Delta ABC$. The next reason because $\Delta AHP \cong \Delta ABC$ and corresponding so $\underline{HP} \| \underline{BC}$ use theorema if there were parallel line with one of triangle side and chop the other two side so comparison length of segment was same, because $\underline{AH} \cong \underline{XY}$ and $\underline{AP} \cong \underline{XZ}$ so $XY/AB = XZ/AC$.

Rd constructed point *K* at $\underline{BC}$ constructed point *L* so $\underline{BK} \cong \underline{YX}$ and $\underline{BL} \cong \underline{YZ}$ and based on side-angle-side theorem were $\underline{BK} \cong \underline{YX}$, $\angle KBL \cong \angle XYZ$, and $\underline{BL} \cong \underline{YZ}$ then $\Delta KBL \cong \Delta XYZ$. Next reason because $\Delta KBL \cong \Delta XYZ$ and use theorem a large number of angle is 180° then $\angle BLK \cong \angle BCA$, next because $\angle BLK \cong \angle BCA$ then $\underline{KL} \| \underline{AC}$. Based on theorem if there was parallel line at one of side triangle then comparison length of segment was comparable. Are $BK/BA = BL/BC$ because $\underline{BK} \cong \underline{XY}$ and $\underline{BL} \cong \underline{YZ}$ then $XY/AB = XZ/AC$. Because $XY/AB = XZ/AC$ and $XY/AB = YZ/BC$ could combine $XY/AB = XZ/AC = YZ/BC$. Because $XY/AB = XZ/AC = YZ/BC$ same comparison and all of the angle were congruent then $\Delta XYZ \sim \Delta ABC$.

Based on commentary above, it can conclude that Rd's reasoning process in stage doing the strategy in prove theorem had been corresponding with rule of logic had been corresponding with coherence truth theorem. This is shown because every statement than making in prove theorem, Rd can give reason base on information in premise or what is known in theorem, requirement congruency of two triangles and theorem be related, that use to prove two triangles are unvarying.

3.4. Looking Back

Reasoning that high and low subjects made on looking back phase to prove the theorem was showing that the three angles of two triangles are congruent and comparison of the length of three corresponding sides of the two triangles are equal then the theorem were proved. Both of subjects felt confident that the conclusions made were right because they were in accordance with the definition of similarity of two triangles and what they did in accordance with the theorem of two similar triangles. Thus the reasoning of the high and low math ability subjects at this looking back phase in the proving the theorem is in conformity with the rules of logic.

Medium mathematical ability subject on looking back phase was confident with his work and because the two triangles have been proved that they were similar, and the reason is in accordance with the theorem of similarity of triangles. The reason subject given was he was confident with his job and because it is in conformity with the definition, but the next answer to say that the job is not appropriate because it does not use the unknown, but he used the area concept of triangle. Since there are answers and reasons given the subject when asked to reflect on the work inconsistent reasoning we can conclude the subject at this stage of rechecking his work in proving the theorem is not in accordance with the rules of logic.

Subject of high math ability has to understand and recognize the concept given time to read the theorem to be proved. He focused on parts of the problem, relating, organizing and analyzing information as well as collecting and combining incorporating the idea of a given situation or problem. He represented mathematical theorem by using pictures and symbols with the relations that exist based on the information which is read on the theorem, using geometry and algebraic manipulation that proving theorem given. If it was connected with the theory of Krulik and Rudnick[14] on the characteristics of reasoning subject ability of mathematical high in proving the theorem starting from understanding phase to looking back the proof can be concluded already in the category of creative thinking, even if the subject does not showed invention for a given problem is a problem of proof. As Polya[12] prove the problem is a problem to indicate whether a statement is true or false, or neither. The problem is a problem that goal proves been determined but the process is required. The main part of this problem is the hypothesis and conclusion of a theorem which must be substantiated.

Subject with medium mathematical ability, in proving the theorem begin to understand the stage until rechecking evidence, showed already understand and recognize the concept given time to read the theorem to be proved. But the subject is less focus on the parts problems in organizing and analyzing information, and do not collect, combine and apply the idea of a given situation or problem. If connected with the theory Krulik and Rudnick[14] on the subject of reasoning characteristic high math ability in proving theorem begin to understand the stage until rechecking the proof can be concluded is not in the category of critical thinking.

If it was connected with the theory of Krulik and Rudnick[14] the subject of mathematical ability is low in proving theorems begin to understand step up to show its proof-check has to understand and recognize the concept given time to read the theorem to be proved. Subject focus on parts of the problem, relating, organizing and analyzing information and collecting, but still having difficulties in combining and applying the idea of a given situation or problem. Subject represented theorem by using pictures and symbols with the relations that exist based on the information which is read on the theorem, using geometry and algebraic manipulation that proving theorems given. If it was connected with the theory Krulik and Rudnick[14] on the characteristics of a subject capable of mathematical reasoning is low in proving theorems already in the category of critical thinking but not yet on the category of creative thinking, because the strategies used less varied.

4. CONCLUSIONS

Based on the questions, the results, and discussion that has been described previously in this study, it can be concluded that the profile of students reasoning in proving theorem of the triangle in terms of differences in mathematical ability is as follows. The Reasoning of students' mathematical ability of high, medium, and low in the understanding theorem is similar. They mention that the premise and the conclusion of the theorem based on logical arguments. When choosing strategy, students with high and medium mathematical ability have similarity in predicting that there are three ways in proving the theorem. Students of low ability chose only a strategy to prove the theorem. At the implementing strategy phase, there are similarities between high and low ability students in proving theorem according to the plans that have been made, although students with high ability used more strategy. A Student with medium mathematical ability did not prove the theorem using the plan mentioned before. The reasoning of students' mathematical ability of high and low when looking back phase are similar, while students' mathematical ability of medium is not similar to the others because there are inconsistent with what has been said. At this phase, there are similar looking back work of students capable of high and low mathematics, while students capable of being which is said to be consistent.

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